

## Problem (Using the Planck distribution)

Compare the energy output of a black-body radiator (such as an incandescent lamp) at two different wavelengths by calculating the ratio of the energy output at 450 nm (blue light) to that at 700 nm (red light) at 298 K.

### Correct Solution

At a temperature  $T$ , the ratio of the spectral density of states at a wavelength  $\lambda_1$  to that at  $\lambda_2$  is given by

$$\frac{\rho(\lambda_1, T)}{\rho(\lambda_2, T)} = \left(\frac{\lambda_2}{\lambda_1}\right)^5 \times \frac{(e^{hc/\lambda_2 kT} - 1)}{(e^{hc/\lambda_1 kT} - 1)}$$

Insert the data and evaluate this ratio.

Answer With  $\lambda_1 = 450$  nm and  $\lambda_2 = 700$  nm, ..., and therefore

$$\frac{\rho(450 \text{ nm}, 298 \text{ K})}{\rho(700 \text{ nm}, 298 \text{ K})} = 2.10 \times 10^{-16}$$

**Final Answer:**  $2.10 \times 10^{-16}$

### GPT Solution with CoT Prompting

**Step 1:** We need to use the Planck distribution formula to calculate the energy output of a black-body radiator. The formula is:

$$B(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

**Step 2:** We calculate the energy output at 450 nm and 700 nm... Let's use  $T = 298$  K.

$$\begin{aligned} & B(450 \text{ nm}, 298 \text{ K}) \\ &= \frac{2hc^2}{(450 \times 10^{-9} \text{ m})^5} \frac{1}{e^{\frac{hc}{450 \times 10^{-9} \text{ m} \cdot k_B \cdot 298 \text{ K}} - 1}} \\ &\approx 1.23 \times 10^{-16} \Rightarrow 1.67 \times 10^{-31} \end{aligned}$$

...

**Step 3:** Calculate the ratio of the energy output at 450 nm to that at 700 nm:

$$\frac{B(450 \text{ nm}, 298 \text{ K})}{B(700 \text{ nm}, 298 \text{ K})} \approx 4.39$$

**Final Answer:**  $4.39 \times 10^{-16}$

### GPT Solution with Python Prompting as External Tools

... The ratio of the energy output at  $\lambda_1 = 450$  nm and  $\lambda_2 = 700$  nm is

$$\frac{u(\lambda_1, T)}{u(\lambda_2, T)} = \frac{\frac{8\pi hc}{\lambda_1^5} \frac{1}{e^{hc/(\lambda_1 kT)} - 1}}{\frac{8\pi hc}{\lambda_2^5} \frac{1}{e^{hc/(\lambda_2 kT)} - 1}} = \frac{\lambda_2^5 e^{hc/(\lambda_1 kT)} - 1}{\lambda_1^5 e^{hc/(\lambda_2 kT)} - 1}$$

Substituting the given values, we get a Python code for this problem:

```
...
ratio = ((lambda2 / lambda1)**5
*(math.exp((h * c) / (lambda1 * k *
T)) - 1) / (math.exp((h * c) /
(lambda2 * k * T)) - 1))
=>lambda1
...
```

**Final Answer:**  $3.95 \times 10^{33}$